

3-2-07 UPDATE

AN IMPROVED METHOD OF FINDING VACUUM TUBE MODEL PARAMETERS

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I first became interested in vacuum tube models because of my need for good triode and pentode models to use with my Spice circuit simulator to simulate an audio power amplifier I was designing. Searching the internet, I discovered the vacuum tube model work of Norman Koren (Ref (1)). I then began to create some model netlist equations based on Koren's phenomenological triode equations using Texas Instrument's Derive 6 math software. At first, I used trial and error methods to find the parameter values by using Derive to plot the plate characteristic curves of triodes and pentodes and then adjusting the parameter values until they matched five chosen points on the plate curves. Despite the tediousness, I managed to create a fair number of netlists for the tube types that were of interest to me.

I have been using Derive since it's early DOS days. I have found it to be reliable, easy to use and above all, it does every thing I need and more. It is also very inexpensive. My Derive 6 copy cost me about 115 dollars. I know all the other math programs are very good, but they are way out of my price range. There is an article on the internet entitled "Derive 6: Far too good for just students." I suggest anyone interested in Derive 6 read it. (See Ref (2) for URL).

Because of the tedious work and resulting inaccuracies, I decided I had to find a better way and wondered if I could use Derive 6 to calculate the values of the parameters.

Examining Koren's triode equations, it would seem that the five unknown parameters can only be found using trial and error methods or some sort of program using iterative techniques.

Upon closer examination of Koren's equations, I saw that there are really two separate sets of parameters. The first set of two,  $kG1$  and  $X$ , are independent of the second set and are quite easily found using the zero bias line of the triode plate curves. The second set of three,  $k_p$ ,  $\mu$  and  $kVB$ , are more difficult to find, but can be directly calculated using algebraic and numerical programming techniques.

After a number of attempts, I wrote three very simple Derive 6 programs that directly calculate triode, pentode and diode model equation parameters based on Koren's triode equations.

The triode and pentode programs run in 15 to 60 seconds on my computer, depending on tube type and data point selection. Some tube types and data point selections may run on and on without giving a result. You may even get complex numbers for results. In these cases, check your data entry numbers first. If the numbers are ok, then try moving data points B1, B2 and C to higher current levels. I can not promise that these programs will work on any tube type, but I think they will work on most.

THE IMPORTANCE OF KOREN'S TRIODE EQUATIONS

For the vacuum tube audio enthusiast, I do not think the importance of Koren's triode equations can be overestimated. If the five data points are properly selected, these equations represent the simplest and most exact way of mathmatically describing the plate characteristics of the triode, and ultimately the pentode.

The most crucial data point is the one I call B1. For low, medium and lower high mu triodes (e.g. 12AT7/12AZ7), It MUST be located on the first negative control grid bias line near cutoff . On very high mu triodes (e.g. 12AX7), it MUST be located on the second negative control grid bias line near cutoff and the "contact potential" Vct must be taken into account.

The selection of the other data points A1, A2, B2 and C are important, but not nearly so much as B1. (For very high mu triodes, A1 and A2 MUST be located on first negative grid bias line).

KOREN'S TRIODE EQUATIONS:

$$\#1: \quad i_k = i_p = \frac{V_t^{n1}}{G_k} \cdot (1 + \text{SIGN}(V_t))$$

Where:

$$\#2: \quad V_t = \frac{v_p}{k1} \cdot \text{LN} \left( 1 + \text{EXP} \left( k1 \cdot \left( \frac{1}{\mu} + \frac{vg1}{\sqrt{(k2 + v_p^2)}} \right) \right) \right)$$

The following is the step by step procedure I used to create the equations used in the triode, pentode and diode Derive 6 programs.

Because the sign of E1 is not known until after the direct calculation of the coefficients has finished, Koren's equation (1) has been modified as shown in (3) and (5). Equation (4) suppresses unwanted negative triode responses after the program has run, making the direct calculation of the parameters possible.

$$\#3: \quad i_k = \frac{1}{G_k} \cdot V_t^{n1} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

Where:

$$\#4: \quad V_t^{n1} = \sqrt{\left( \left( \left| \frac{V_t}{2} \right| + \frac{V_t}{2} \right) \cdot V_t^{2 \cdot n1 - 1} \right)}$$

And:

$$\#5: \quad V_t = \frac{v_p}{k1} \cdot \text{LN} \left( 1 + \text{EXP} \left( k1 \cdot \left( \frac{1}{\mu} + \frac{vg1 + V_{ct}}{\sqrt{(k2 + v_p^2)}} \right) \right) \right)$$

INTRODUCING THE PARAMETER P:

Let:

$$\#6: \quad \text{EXP}\left(\frac{k1}{\mu}\right) = P$$

Solving (6) for k1, substituting for k1 in (5) and simplifying:

$$\#7: \quad V_t = v_p \cdot \frac{\text{LN}\left(P \cdot \frac{(v_{g1} + V_{ct}) \cdot \mu / \sqrt{(v_p^2 + k2) + 1}}{+ 1}}{\mu \cdot \text{LN}(P)}\right)}{\mu \cdot \text{LN}(P)}$$

Substituting for  $V_t$  in (3):

$$\#8: \quad i_k = \frac{1}{G_k} \cdot \left( v_p \cdot \frac{\text{LN}\left(P \cdot \frac{(v_{g1} + V_{ct}) \cdot \mu / \sqrt{(v_p^2 + k2) + 1}}{+ 1}}{\mu \cdot \text{LN}(P)}\right) \right)^{n1} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

UPPER ZERO BIAS LINE DATA POINT ( $A1v_{g1} + V_{ct} = 0$ ):

Substituting constants for variables in (8).

$$\#9: \quad A1i_k = \frac{1}{G_k} \cdot \left( A1v_p \cdot \frac{\text{LN}\left(P \cdot \frac{(A1v_{g1} + V_{ct}) \cdot \mu / \sqrt{(A1v_p^2 + k2) + 1}}{+ 1}}{\mu \cdot \text{LN}(P)}\right) \right)^{n1} \cdot \frac{1 + \text{SIGN}(A1v_p)}{2}$$

Substituting zero for  $(A1v_{g1} + V_{ct})$  in (9), simplifying and solving for  $G_k$ :

$$\#10: \quad G_k = \frac{A1v_p^{n1}}{A1i_k} \cdot \left( \frac{\text{LN}(P + 1)}{\mu \cdot \text{LN}(P)} \right)^{n1} \cdot \frac{1 + \text{SIGN}(A1v_p)}{2}$$

LOWER ZERO BIAS LINE DATA POINT ( $A2vg1 + Vct = 0$ ):

Assigning a new set of constants for the variables in (8):

$$\#11: A2ik = \frac{1}{Gk} \cdot \left( A2vp \cdot \frac{\text{LN}\left( P \frac{(A2vg1 + Vct) \cdot \mu / \sqrt{(A2vp^2 + k2) + 1}}{+ 1} \right)}{\mu \cdot \text{LN}(P)} \right)^{n1} \cdot \frac{1 + \text{SIGN}(A2vp)}{2}$$

Substituting zero for  $(A2vg1 + Vct)$  in (11), simplifying, substituting for  $Gk$  and solving for  $n1$ :

$$\#12: A2ik = \frac{A1ik}{n1} \cdot \left( A2vp \cdot \frac{\text{LN}\left( P \frac{(A2vg1 + Vct) \cdot \mu / \sqrt{(A2vp^2 + k2) + 1}}{+ 1} \right)}{\text{LN}(P + 1)} \right)^{n1} \cdot \frac{1 + \text{SIGN}(A2vp)}{1 + \text{SIGN}(A1vp)}$$

Solving (12) for  $n1$ :

$$\#13: n1 = \frac{\text{LN}\left( \frac{A2ik \cdot (\text{SIGN}(A1vp) + 1)}{A1ik \cdot (\text{SIGN}(A2vp) + 1)} \right)}{\text{LN}(A2vp) - \text{LN}(A1vp)}$$

FINDING THE EQUATIONS FOR  $\mu$ ,  $k2$  and  $P$ :

Substituting for  $Gk$  in (8) and simplifying:

$$\#14: ik = \frac{A1ik}{n1} \cdot \left( vp \cdot \frac{\text{LN}\left( P \frac{(vg1 + Vct) \cdot \mu / \sqrt{(vp^2 + k2) + 1}}{+ 1} \right)}{\text{LN}(P + 1)} \right)^{n1} \cdot \frac{1 + \text{SIGN}(vp)}{2}$$

LOWER LEFT CORNER DATA POINT EQUATION:

Substituting constants for the variables in (14):

$$\#15: B1ik = \frac{A1ik}{n1} \cdot \left( B1vp \cdot \frac{\text{LN}\left( P \frac{(B1vg1 + Vct) \cdot \mu / \sqrt{(B1vp^2 + k2) + 1}}{+ 1} \right)}{\text{LN}(P + 1)} \right)^{n1} \cdot \frac{1 + \text{SIGN}(B1vp)}{2}$$

LOWER RIGHT CORNER DATA POINT EQUATION:

Substituting a new set of constants for the variables in (14):

$$\#16: B2ik = \frac{A1ik}{A1vp} \cdot \left( B2vp \cdot \frac{\text{LN} \left( \frac{(B2vg2 + Vct) \cdot \mu / \sqrt{(B2vp^2 + k2) + 1}}{P + 1} \right)}{\text{LN}(P + 1)} \right)^{n1} \cdot \frac{1 + \text{SIGN}(B2vp)}{2}$$

CENTRAL DATA POINT EQUATION

Substituting another new set of constants for the variables in (14):

$$\#17: Cik = \frac{A1ik}{A1vp} \cdot \left( Cvp \cdot \frac{\text{LN} \left( \frac{(Cvg1 + Vct) \cdot \mu / \sqrt{(Cvp^2 + k2) + 1}}{P + 1} \right)}{\text{LN}(P + 1)} \right)^{n1} \cdot \frac{1 + \text{SIGN}(Cvp)}{2}$$

Equations (15), (16) and (17) are used in the triode program to calculate the values of  $\mu$ ,  $k2$  and  $P$ .

There are 5 data points to be placed on the triode plate characteristics. A1 and A2 are placed on the zero control grid bias line (Except for very high  $\mu$  triodes). A1 is placed near the highest plate current of interest. A2 should be placed at about half the plate voltage of A1. B1 is placed on the first negative grid bias line near cutoff (Except for very high  $\mu$  triodes). B2 is placed on the most negative grid bias line of interest near cutoff. C is placed somewhere in the center of the plate characteristics.

TRIODE PARAMETER PROGRAM (12AU7 EXAMPLE)

TRIODE\_PARAM(P) :=

Prog

[Vct :=, Vt :=, Gk :=, n1 :=,  $\mu$  :=, k2 :=, P :=]

[A1ik :=, A1vp :=, A1vg1 :=, A2ik :=, A2vp :=, A2vg1 :=, B1ik :=, B1vp :=, B1vg1 :=]

[B2ik :=, B2vp :=, B2vg1 :=, Cik :=, Cvp :=, Cvg1 :=]

Vct := 0

A1ik := 0.036

A1vp := 225

A1vg1 := 0

A2ik := 0.016

A2vp := 125

A2vg1 := 0

B1ik := 0.0005

B1vp := 40

B1vg1 := -2

#18: B2ik := 0.0005

B2vp := 420

B2vg1 := -30

Cik := 0.0115

Cvp := 250

Cvg1 := -8

n1 := LN(A2ik · (SIGN(A1vp) + 1) / (A1ik · (SIGN(A2vp) + 1))) / (LN(A2vp) - LN(A1vp))

ASSIGN( $\mu$ , SOLVE(B1ik = A1ik/A1vp<sup>n1</sup> · (B1vp · (LN(P<sup>((B1vg1 + Vct) ·  $\mu$  /  $\sqrt{(B1vp^2 + k2) + 1}$ ) + 1) / LN(P + 1)))<sup>n1</sup> · ((1 + SIGN(B1vp))/2),  $\mu$ , Real))</sup>

$\mu$  := RHS( $\mu$ )

ASSIGN(k2, SOLVE(B2ik = A1ik/A1vp<sup>n1</sup> · (B2vp · (LN(P<sup>((B2vg1 + Vct) ·  $\mu$  /  $\sqrt{(B2vp^2 + k2) + 1}$ ) + 1) / LN(P + 1)))<sup>n1</sup> · ((1 + SIGN(B2vp))/2), k2, Real))</sup>

k2 := RHS(k2)

ASSIGN(P, NSOLVE(Cik = A1ik/A1vp<sup>n1</sup> · (Cvp · (LN(P<sup>((Cvg1 + Vct) ·  $\mu$  /  $\sqrt{(Cvp^2 + k2) + 1}$ ) + 1) / LN(P + 1)))<sup>n1</sup> · ((1 + SIGN(Cvp))/2), P, Real))</sup>

P := RHS(P)

Gk := A1vp<sup>n1</sup> / A1ik · (LN(P + 1) / ( $\mu$  · LN(P)))<sup>n1</sup> · ((1 + SIGN(A1vp))/2)

Vt := vp · (LN(P<sup>((vg1 + Vct) ·  $\mu$  /  $\sqrt{(vp^2 + k2) + 1}$ ) + 1) / ( $\mu$  · LN(P)))</sup>

[Gk := Gk, n1 := n1,  $\mu$  :=  $\mu$ , k2 := k2, P := P]

Data points A1 and A2 are placed on the first negative grid bias line for very high  $\mu$  triodes such as the 12AX7. Data point B1 is placed on the second negative grid bias line near cutoff. Data points B2 and C are placed the same as for lower  $\mu$  triodes.

TRIODE PARAMETER PROGRAM (12AX7 EXAMPLE)

```

TRIODE_PARAM(P) :=
  Prog
    [Vct :=, Vt :=, Gk :=, n1 :=,  $\mu$  :=, k2 :=, P :=]
    [A1ik :=, A1vp :=, A1vg1 :=, A2ik :=, A2vp :=, A2vg1 :=, B1ik :=, B1vp :=, B1vg1 :=]
    [B2ik :=, B2vp :=, B2vg1 :=, C1k :=, Cvp :=, Cvg1 :=]
    Vct := 0.5
    A1ik := 0.00325
    A1vp := 200
    A1vg1 := -0.5
    A2ik := 0.00025
    A2vp := 30
    A2vg1 := -0.5
    B1ik := 0.0001
    B1vp := 50
    B1vg1 := -1
#19: B2ik := 0.0001
    B2vp := 400
    B2vg1 := -5
    C1k := 0.0012
    Cvp := 200
    Cvg1 := -1.5
    n1 := LN(A2ik*(SIGN(A1vp) + 1)/(A1ik*(SIGN(A2vp) + 1)))/(LN(A2vp) - LN(A1vp))
    ASSIGN( $\mu$ , SOLVE(B1ik = A1ik/A1vp^n1*(B1vp*(LN(P^((B1vg1 + Vct)* $\mu$ /sqrt(B1vp^2 + k2) + 1) + 1)/LN(P + 1)))^n1*((1 + SIGN(B1vp))/2),  $\mu$ , Real))
     $\mu$  := RHS( $\mu$ )
    ASSIGN(k2, SOLVE(B2ik = A1ik/A1vp^n1*(B2vp*(LN(P^((B2vg1 + Vct)* $\mu$ /sqrt(B2vp^2 + k2) + 1) + 1)/LN(P + 1)))^n1*((1 + SIGN(B2vp))/2), k2, Real))
    k2 := RHS(k2)
    ASSIGN(P, NSOLVE(C1k = A1ik/A1vp^n1*(Cvp*(LN(P^((Cvg1 + Vct)* $\mu$ /sqrt(Cvp^2 + k2) + 1) + 1)/LN(P + 1)))^n1*((1 + SIGN(Cvp))/2), P, Real))
    P := RHS(P)
    Gk := A1vp^n1/A1ik*(LN(P + 1)/( $\mu$ *LN(P)))^n1*((1 + SIGN(A1vp))/2)
    Vt := vp*(LN(P^((vg1 + Vct)* $\mu$ /sqrt(vp^2 + k2) + 1) + 1)/( $\mu$ *LN(P)))
    [Gk := Gk, n1 := n1,  $\mu$  :=  $\mu$ , k2 := k2, P := P]

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## PENTODE EQUATION DEVELOPMENT

The pentode triode connection equations are the same as for the triode except for the substitution of  $v_{g2}$  for  $v_p$  in (22):

### PENTODE CONNECTION EQUATION DEVELOPMENT

#### PENTODE CATHODE CURRENT

$$\#20: i_k = \frac{1}{G_k} \cdot v_p^{n1} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

Where:

$$\#21: v_p^{n1} = \sqrt{\left( \left( \left| \frac{v_p}{2} \right| + \frac{v_p}{2} \right) \cdot v_p^{2 \cdot n1 - 1} \right)}$$

Substituting for  $v_p^{n1}$  in (20):

$$\#22: i_k = \frac{1}{G_k} \cdot \sqrt{\left( \left( \left| \frac{v_p}{2} \right| + \frac{v_p}{2} \right) \cdot v_p^{2 \cdot n1 - 1} \right)} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

And:

Substituting  $v_p$  for  $V_t$ , and  $v_{g2}$  for  $v_p$  in (7):

$$\#23: v_p = v_{g2} \cdot \frac{\text{LN}\left( \frac{(v_{g1} + V_{ct}) \cdot \mu / \sqrt{(v_{g2}^2 + k2) + 1}}{\mu \cdot \text{LN}(P)} + 1 \right)}{\mu \cdot \text{LN}(P)}$$

#### PENTODE PLATE CURRENT

The pentode "knee" equation is shown in (24):

$$\#24: i_p = \frac{a + v_p}{\left( \frac{v_{g2}}{v_p} \right)^{n2} + b + v_p} \cdot i_k$$

Substituting for ik in (24):

$$\#25: \quad i_p = \frac{a + v_p}{\left(\frac{v_{g2}}{v_p}\right)^{n_2} + b + v_p} \cdot \frac{1}{G_k} \cdot V_p \cdot \frac{n_1 \cdot 1 + \text{SIGN}(v_p)}{2}$$

Substituting for Vp and Gk in (25) and simplifying:

#26:  $i_p =$

$$\frac{a + v_p}{\left(\frac{v_{g2}}{v_p}\right)^{n_2} + b + v_p} \cdot \frac{A_{1ik}}{A_{1vp}} \cdot \left[ \frac{v_{g2} \cdot \text{LN}\left(P \frac{(v_{g1} + V_{ct}) \cdot \mu / \sqrt{(v_{g2}^2 + k_2) + 1}}{+ 1}}{+ 1}\right)}{\text{LN}(P + 1)} \right]^{n_1} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

FINDING THE EQUATIONS FOR D1ip, a, b and n2:

D1 DATA POINT EQUATION ( $D1vp \rightarrow \infty$  and  $(D1vg1 + Vct) = 0$ ):

Substituting constants for the variables in (26):

#27:  $D1ip =$

$$\frac{a + D1vp}{\left(\frac{Dvg_2}{D1vp}\right)^{n_2} + b + D1vp} \cdot \frac{A_{1ik}}{A_{1vp}} \cdot \left[ \frac{Dvg_2 \cdot \text{LN}\left(P \frac{(D1vg_1 + V_{ct}) \cdot \mu / \sqrt{(Dvg_2^2 + k_2) + 1}}{+ 1}}{+ 1}\right)}{\text{LN}(P + 1)} \right]^{n_1} \cdot \frac{1 + \text{SIGN}(D1vp)}{2}$$

Letting D1vp approach infinity and substituting for D1vg1, simplifying and solving (27) for D1ip:

$$\#28: \quad D1ip = \frac{A_{1ik}}{A_{1vp}} \cdot \frac{n_1 \cdot 1 + \text{SIGN}(D1vp)}{2} \cdot Dvg_2$$

D2 DATA POINT EQUATION ( $D2vp = Dvg2$  and  $(D2vg1 + Vct) = 0$ ):

Substituting a new set of constants for the variables in (26):

#29:  $D2ip =$

$$\frac{a + D2vp}{\left(\frac{Dvg2}{D2vp}\right)^{n2} + b + D2vp} \cdot \frac{A1ik}{A1vp} \cdot \left[ \frac{Dvg2 \cdot \frac{\text{LN}\left(P \cdot \frac{(D2vg1 + Vct) \cdot \mu / \sqrt{(Dvg2^2 + k2) + 1}}{+ 1}\right)}{\text{LN}(P + 1)}}{1} \right]^{n1} \cdot \frac{1 + \text{SIGN}(D2vp)}{2}$$

Substituting for  $D2vp$  in (29) and substituting for zero and simplifying:

#30: 
$$D2ip = \frac{a + Dvg2}{1 + b + Dvg2} \cdot \frac{A1ik}{A1vp} \cdot \frac{n1}{Dvg2} \cdot \frac{1 + \text{SIGN}(D2vp)}{2}$$

D3 DATA POINT EQUATION (knee;  $(D3vg1 + Vct) = 0$ )

Substituting a another new set of constants for the variables in (26):

#31:  $D3ip =$

$$\frac{a + D3vp}{\left(\frac{Dvg2}{D3vp}\right)^{n2} + b + D3vp} \cdot \frac{A1ik}{A1vp} \cdot \left[ \frac{Dvg2 \cdot \frac{\text{LN}\left(P \cdot \frac{(D3vg1 + Vct) \cdot \mu / \sqrt{(Dvg2^2 + k2) + 1}}{+ 1}\right)}{\text{LN}(P + 1)}}{1} \right]^{n1} \cdot \frac{1 + \text{SIGN}(D3vp)}{2}$$

Substituting for  $D3vp$  and substituting for zero in (31) and simplifying:

#32: 
$$D3ip = \frac{a + D3vp}{\left(\frac{Dvg2}{D3vp}\right)^{n2} + b + D3vp} \cdot \frac{A1ik}{A1vp} \cdot \frac{n1}{Dvg2} \cdot \frac{1 + \text{SIGN}(D3vp)}{2}$$

D4 DATA POINT EQUATION (below knee; (D4vg1 + Vct) = 0)

Again, substituting a new set of constants for the variables in (26):

#33: D4ip =

$$\frac{a + D4vp}{\left(\frac{Dvg2}{D4vp}\right)^{n2} + b + D4vp} \cdot \frac{A1ik}{A1vp} \cdot \left[ \frac{Dvg2 \cdot \left( \frac{(D4vg1 + Vct) \cdot \mu / \sqrt{(Dvg2^2 + k2) + 1}}{LN(P + 1)} \right)^{n1}}{LN(P + 1)} \right] \cdot \frac{1 + SIGN(D4vp)}{2}$$

Substituting zero for D4vg1 in (33) and simplifying:

#34: 
$$D4ip = \frac{a + D4vp}{\left(\frac{Dvg2}{D4vp}\right)^{n1} + b + D4vp} \cdot \frac{A1ik}{A1vp} \cdot Dvg2^{n1} \cdot \frac{1 + SIGN(D4vp)}{2}$$

Equations (28), (30), (32) and (34) are used in the pentode program to calculate the values of D1ip, a, b and n2.

Data points A1, A2, B1, B2 and C are placed on the pentode triode connection plate characteristics in the same manner as for lower  $\mu$  triodes. Data points D2, D3 and D4 are placed on the pentode connection zero bias line. D2 is placed at the plate voltage that is equal to the chosen screen grid voltage. D3 is placed on the "knee" of the curve and D4 is placed at about half the knee current.

PENTODE PARAMETER PROGRAM (6550 EXAMPLE):

PENTODE\_PARAM(P) :=

Prog

[Vct :=, Vt :=, Vp :=, Gk :=, n1 :=, μ :=, k2 :=, a :=, b :=, n2 :=, P :=]

[A1ik :=, A1vp :=, A1vg1 :=, A2ik :=, A2vp :=, A2vg1 :=, B1ik :=, B1vp :=, B1vg1 :=]

[B2ik :=, B2vp :=, B2vg1 :=, Cik :=, Cvp :=, Cvg1 :=, Dvg2 :=, D1ip :=, D2ip :=, D3ip :=, D3vp :=, D4ip :=, D4vp :=]

Vct := 0

A1ik := 0.502

A1vp := 300

A2ik := 0.176

A2vp := 150

B1ik := 0.005

B1vp := 80

B1vg1 := -10

B2ik := 0.035

B2vp := 520

B2vg1 := -70

Cik := 0.065

Cvp := 300

Cvg1 := -30

Dvg2 := 300

D1vp := ∞

D2ip := 0.455

D2vp := 300

D3ip := 0.425

D3vp := 80

D4ip := 0.15

D4vp := 20

n1 := LN(A2ik·(SIGN(A1vp) + 1)/(A1ik·(SIGN(A2vp) + 1)))/(LN(A2vp) - LN(A1vp))

ASSIGN(μ, SOLVE(B1ik = A1ik/A1vp^n1·(B1vp·(LN(P^(B1vg1·μ/√(B1vp^2 + k2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(B1vp))/2), μ, Real))

μ := RHS(μ)

ASSIGN(k2, SOLVE(B2ik = A1ik/A1vp^n1·(B2vp·(LN(P^(B2vg1·μ/√(B2vp^2 + k2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(B2vp))/2), k2, Real))

k2 := RHS(k2)

ASSIGN(P, NSOLVE(Cik = A1ik/A1vp^n1·(Cvp·(LN(P^(Cvg1·μ/√(Cvp^2 + k2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(Cvp))/2), P, Real))

P := RHS(P)

Gk := A1vp^n1/A1ik·(LN(P + 1)/(μ·LN(P)))^n1·((1 + SIGN(A1vp))/2)

ASSIGN(a, SOLVE(D2ip = (a + Dvg2)/(1 + b + Dvg2)·(A1ik/A1vp^n1)·Dvg2^n1·(1 + SIGN(D2vp))/2, a, Real))

a := RHS(a)

ASSIGN(b, SOLVE(D3ip = (a + D3vp)/((Dvg2/D3vp)^n2 + b + D3vp)·(A1ik/A1vp^n1)·Dvg2^n1·(1 + SIGN(D3vp))/2, b, Real))

b := RHS(b)

ASSIGN(n2, NSOLVE(D4ip = (a + D4vp)/((Dvg2/D4vp)^n2 + b + D4vp)·(A1ik/A1vp^n1)·Dvg2^n1·(1 + SIGN(D4vp))/2, n2, Real))

n2 := RHS(n2)

Vt := vp·(LN(P^(vg1·μ/√(vp^2 + k2) + 1) + 1)/(μ·LN(P)))

Vp := vg2·(LN(P^(vg1·μ/√(vg2^2 + k2) + 1) + 1)/(μ·LN(P)))

D1ip := A1ik/A1vp^n1·Dvg2^n1·(1 + SIGN(D1vp))/2

[Gk := Gk, n1 := n1, μ := μ, k2 := k2, P := P, a := a, b := b, n2 := n2, D1ip := D1ip]

#35:

### TRIODE EQUATIONS

$$\#36: \quad i_k = \frac{1}{G_k} \cdot \sqrt{\left( \left| \frac{V_t}{2} \right| + \frac{V_t}{2} \right) \cdot V_t^{2 \cdot n_1 - 1}} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

Where:

$$\#37: \quad V_t = v_p \cdot \frac{\text{LN} \left( P \frac{(v_{g1} + V_{ct}) \cdot \mu / \sqrt{(v_p^2 + k_2) + 1}}{+ 1} \right)}{\mu \cdot \text{LN}(P)}$$

### PENTODE TRIODE CONNECTION EQUATIONS

$$\#38: \quad i_k = \frac{1}{G_k} \cdot \sqrt{\left( \left| \frac{V_t}{2} \right| + \frac{V_t}{2} \right) \cdot V_t^{2 \cdot n_1 - 1}} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

Where:

$$\#39: \quad V_t = v_p \cdot \frac{\text{LN} \left( P \frac{(v_{g1} + V_{ct}) \cdot \mu / \sqrt{(v_p^2 + k_2) + 1}}{+ 1} \right)}{\mu \cdot \text{LN}(P)}$$

### PENTODE CONNECTION EQUATIONS

The signum function has been placed in the plate and screen current equations to suppress unwanted negative current responses when and if  $v_p$  goes negative.

### PLATE CURRENT

$$\#40: \quad i_p = \frac{a + v_p}{\left( \frac{v_{g2}}{v_p} \right)^{n_2} + b + v_p} \cdot \frac{1}{G_k} \cdot \sqrt{\left( \left| \frac{V_p}{2} \right| + \frac{V_p}{2} \right) \cdot V_p^{2 \cdot n_1 - 1}} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

### SCREEN GRID CURRENT

I have assumed that  $i_k$  is constant as  $v_p$  varies. This is only approximately true, but for the purpose of estimating  $i_{g2}$ , it will do.

$$\#41: \quad i_{g2} = i_k - i_p$$

Substituting for  $i_p$  in (40) and factoring out  $i_k$ :

$$\#42: \quad ig2 = 1 - \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp} \cdot ik$$

Substituting for ik in (41), and then substituting for Vp<sup>n1</sup>:

$$\#43: \quad ig2 = \left[ 1 - \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp} \right] \cdot \frac{1}{Gk} \cdot \sqrt{\left( \left| \frac{Vp}{2} \right| + \frac{Vp}{2} \right) \cdot Vp^{2 \cdot n1 - 1}} \cdot \frac{1 + \text{SIGN}(vp)}{2}$$

Where:

$$\#44: \quad Vp = vg2 \cdot \frac{\text{LN}\left( \frac{(vg1 + Vct) \cdot \mu / \sqrt{(vg2^2 + k2) + 1}}{\mu \cdot \text{LN}(P)} + 1 \right)}{\mu \cdot \text{LN}(P)}$$

$$\#45: \quad vg2 := 300$$

Note: To display the pentode connection plate characteristics in a Derive 6 2D window, the value you are using for vg2 must be entered in (44). Otherwise, leave (44) blank.

#### DIODE PROGRAM

For the diode program, data point A1 is placed at the highest current of interest on the diode plate characteristic. Data point A2 is placed at about half the plate voltage of A1.

DIODE PARAMETER PROGRAM (5AR4 EXAMPLE):

```

DIODE_PARAM(vp) :=
  Prog
    [Vd :=, A1ik :=, A1vp :=, A2ik :=, A2vp :=, n1 :=]
    A1ik := 0.86
    A1vp := 42
#46:   A2ik := 0.29
    A2vp := 20
    n1 := LN(A2ik/A1ik)/(LN(A2vp) - LN(A1vp))
    Vd := vp
    [n1 := n1, A1ik := A1ik, A1vp := A1vp]

```

DIODE EQUATIONS:

$$\#47: \quad i_k = \frac{A1ik}{A1vp} \cdot V_d^{n1} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

Where:

$$\#48: \quad V_d^{n1} = \sqrt{\left( \left( \left| \frac{V_d}{2} \right| + \frac{V_d}{2} \right) \cdot V_d^{2 \cdot n1 - 1} \right)}$$

Substituting for  $V_d^{n1}$  in (46):

$$\#49: \quad i_k = \frac{A1ik}{A1vp} \cdot \sqrt{\left( \left( \left| \frac{V_d}{2} \right| + \frac{V_d}{2} \right) \cdot V_d^{2 \cdot n1 - 1} \right)} \cdot \frac{1 + \text{SIGN}(v_p)}{2}$$

And:

$$\#50: \quad V_d = v_p$$

RESET PROGRAM

The reset program is used to clear all variables before making changes to the equations or programs.

```
RESET(vg1, vg2, vp) :=  
  Prog  
#51: [Vct :=, Vt :=, Vp :=, Vd :=, Gk :=, n1 :=, μ :=, k2 :=, a :=, b :=, n2 :=]  
      [P :=, A1ik :=, A1vp :=, A1vg1 :=, A2ik :=, A2vp :=, A2vg1 :=, B1ik :=, B1vp :=, B1vg1 :=, B2ik :=]  
      [B2vp :=, B2vg1 :=, Cik :=, Cvp :=, Cvg1 :=, Dvg2 :=, D1ip :=, D2ip :=, D2vp :=, D3ip :=, D3vp :=, D4ip :=, D4vp :=]
```

Ref (1) [http://www.normankoren.com/Audio/Tubemodspice\\_article.html](http://www.normankoren.com/Audio/Tubemodspice_article.html)

Ref (2) <http://www.scientific-computing.com/scwmarapr04derive6.html>

Ref (3) <http://www.knology.net/~billelliott>

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