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MORE ABOUT FINDING SPICE VACUUM TUBE MODEL PARAMETERS

By Bill Elliott

This work is based on the work of Norman Koren (Ref 1) and his new equation (circa 2000) for vacuum tube triodes (shown below). It is also a continuation of a previous article (Ref 2) I wrote on the same subject. It shows the procedure I used to write a Derive 6 program for directly calulating the parameter values for pentode vacuum tube spice models. It is a step by step procedure that can be used as a guide for writing similar programs using other math software. It also shows the development of a new equation for pentodes based on Koren's new triode equation.

It is written with Derive 6 software rather than some other math software because that is what I have.

Koren's Triode Equations:

#1: $$Ip = \frac{E1^{X}}{kG1}\cdot(1 + SIGN(E1))$$

Where:

#2: $$E1 = \frac{Ep}{kp}\cdot LN\left(1 + EXP\left(kp\cdot\left(\frac{1}{\mu} + \frac{EG}{\sqrt{(kVB + Ep^{2})}}\right)\right)\right)$$

THE IMPORTANCE OF KOREN'S TRIODE EQUATIONS

For the vacuum tube audio enthusiast, I do not think the importance of Koren's triode equations can be overestimated. If the five data points are properly selected, these equations represent the simplest and most exact way of mathmatically describing the plate characteristics of the triode, and ultimately the pentode.

Koren's equation (2) can be looked upon as a conversion of the Langmuir-Childs equation as shown below. I call this Koren's conversion.

Langmuir-Childs triode equation:

#3: $$Ip = \frac{1}{kG1}\cdot\left(EG + \frac{Ep}{\mu}\right)^{X}\cdot(1 + SIGN(E1))$$

Factoring out Ep:

#4: $$Ip = \frac{1}{kG1}\cdot\left(Ep\cdot\left(\frac{EG}{Ep} + \frac{1}{\mu}\right)\right)^{X}\cdot(1 + SIGN(E1))$$

Applying Koren's conversion to (4):

#5: $$Ip = \frac{1}{kG1}\cdot\left(\frac{Ep}{kp}\cdot LN\left(1 + EXP\left(kp\cdot\left(\frac{EG}{\sqrt{(kVB + Ep^{2})}} + \frac{1}{\mu}\right)\right)\right)\right)^{X}\cdot(1 + SIGN(E1))$$

Koren's conversion can also applied to the Langmuir-Childs pentode equation as shown later.

STEP BY STEP PROCEDURE:

Because the sign of E1 is not known until after the direct calculation of the coefficients has finished, Koren's equation (1) has been modified by replacing E1 with vp. Since this only suppresses unwanted negative voltage outputs, an equation of the type shown in (83) is used to suppress unwanted negative current outputs.

To go through the step by step procedure, first, simplify the RESET(P) command (6) (DFW format only).

#6: RESET(P)

LANGMUIR-CHILDS TRIODE EQUATION:

#7: $$ik = \frac{1}{Gk}\cdot\left(vg1 + Vct + \frac{vp}{\mu t}\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Factoring out vp in (7):

#8: $$ik = \frac{1}{Gk}\cdot\left(vp\cdot\left(\frac{vg1 + Vct}{vp} + \frac{1}{\mu t}\right)\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Applying Koren's conversion to (8):

$$\#9: \quad ik = \frac{1}{Gk}\cdot\left(\frac{vp}{k1}\cdot LN\left(1 + EXP\left(k1\cdot\left(\frac{vg1 + Vct}{\sqrt{(k2 + vp^2)}} + \frac{1}{\mu t}\right)\right)\right)\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Introduction of the parameter P:

Let:

$$\#10: \quad EXP\left(\frac{k1}{\mu t}\right) = P$$

Solving (10) for k1:

$$\#11: \quad k1 = \mu t\cdot LN(P)$$

Substituting for k1 in (9):

$$\#12: \quad ik = \frac{1}{Gk}\cdot\left(\frac{vp}{\mu t\cdot LN(P)}\cdot LN\left(1 + EXP\left(\mu t\cdot LN(P)\cdot\left(\frac{vg1 + Vct}{\sqrt{(k2 + vp^2)}} + \frac{1}{\mu t}\right)\right)\right)\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Simplifying (12) yields equation (13):

$$\#13: \quad ik = \frac{1}{Gk}\cdot\left(\frac{vp}{\mu t\cdot LN(P)}\cdot LN\left(P^{(\sqrt{(k2 + vp^2)} + (vg1 + Vct)\cdot\mu t)/\sqrt{(k2 + vp^2)}} + 1\right)\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Expanding (13) yields (14):

$$\#14: \quad ik = \frac{1}{Gk}\cdot\left(\frac{vp}{\mu t\cdot LN(P)}\cdot LN\left(P^{(vg1 + Vct)\cdot\mu t/\sqrt{(vp^2 + k2)} + 1} + 1\right)\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Rearranging terms in (14) yields (15):

$$\#15: \quad ik = \frac{1}{Gk}\cdot\left(\frac{vp}{\mu t}\cdot\frac{LN\left(P^{(vg1 + Vct)\cdot\mu t/\sqrt{(k2 + vp^2)} + 1} + 1\right)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

LANGMUIR-CHILDS PENTODE EQUATION:

$$\#16: \quad ik = \frac{1}{Gk}\cdot\left(vg1 + Vct + \frac{vg2}{\mu g2} + \frac{vp}{\mu p}\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Factoring out vg2 in (16):

$$\#17: \quad ik = \frac{1}{Gk}\cdot\left(vg2\cdot\left(\frac{vg1 + Vct}{vg2} + \frac{1}{\mu g2} + \frac{vp}{vg2\cdot\mu p}\right)\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Applying Koren's conversion to (17):

$$\#18: \quad ik = \frac{1}{Gk}\cdot\left(\frac{vg2}{k1}\cdot LN\left(1 + EXP\left(k1\cdot\left(\frac{vg1 + Vct}{\sqrt{(k2 + vg2^2)}} + \frac{1}{\mu g2} + \frac{vp}{vg2\cdot\mu p}\right)\right)\right)\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Substituting for k1 in (18):

$$\#19: \quad ik = \frac{1}{Gk}\cdot\left(\frac{vg2}{\mu t\cdot LN(P)}\cdot LN\left(1 + EXP\left(\mu t\cdot LN(P)\cdot\left(\frac{vg1 + Vct}{\sqrt{(k2 + vg2^2)}} + \frac{1}{\mu g2} + \frac{vp}{vg2\cdot\mu p}\right)\right)\right)\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

When vg2 equals vp, it can be shown that:

$$\#20: \quad \frac{1}{\mu g2} + \frac{1}{\mu p} = \frac{1}{\mu t}$$

Solving (20) for μg2:

$$\#21: \quad \mu g2 = \frac{\mu p\cdot\mu t}{\mu p - \mu t}$$

Substituting for μg2 in (19) and simplifying the results as in (13) through (15) yields the pentode equation for cathode current

:

#22:

$$ik = \frac{1}{Gk}\cdot\left(\frac{vg2}{\mu t}\cdot\frac{LN\left(P^{(vg1 + Vct)\cdot\mu t/\sqrt{(k2 + vg2^2)} + vp\cdot\mu t/(vg2\cdot\mu p) + (\mu p - \mu t)/\mu p} + 1\right)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Substituting constants for the variables in (22):

#23:

$$Aik = \frac{1}{Gk}\cdot\left(\frac{Avg2}{\mu t}\cdot\frac{LN\left(P^{(Avg1 + Vct)\cdot\mu t/\sqrt{(k2 + Avg2^2)} + Avp\cdot\mu t/(Avg2\cdot\mu p) + (\mu p - \mu t)/\mu p} + 1\right)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(Avp)}{2}$$

Because we are interested only in positive values of vp, ((1 + SIGN(Avp))/2) can be simplifyied to one:

#24:

$$Aik = \frac{1}{Gk}\cdot\left(\frac{Avg2}{\mu t}\cdot\frac{LN\left(P^{(Avg1 + Vct)\cdot\mu t/\sqrt{(k2 + Avg2^2)} + Avp\cdot\mu t/(Avg2\cdot\mu p) + (\mu p - \mu t)/\mu p} + 1\right)}{LN(P)}\right)^{n1}$$

Solving (24) for μp:

#25:

$$\mu p = \mathrm{IF}\left(n1 \le -1 \lor Aik\cdot Gk > 0 \lor n1 \ge 1,\ \mathrm{IF}\left(-\pi < \mathrm{IM}\left(\frac{(\mu t\cdot(Aik\cdot Gk))^{1/n1}\cdot \mathrm{LN}(P)}{Avg2}\right) \le \pi,\ \frac{\mu t\cdot(Avp - Avg2)\cdot \mathrm{LN}(P)}{Avg2\cdot \mathrm{LN}\left(P^{-\,(Avg1\cdot\mu t + \sqrt{(Avg2^2 + k2)} + Vct\cdot\mu t)/\sqrt{(Avg2^2 + k2)}}\cdot\left(P^{(\mu t\cdot(Aik\cdot Gk))^{1/n1}/Avg2} - 1\right)\right)}\right)\right)$$

Evaluating the IF statments:

#26:

$$\mu p = \mathrm{IF}\left(\text{false} \lor \text{true} \lor \text{true},\ \mathrm{IF}\left(\text{true},\ \frac{\mu t\cdot(Avp - Avg2)\cdot \mathrm{LN}(P)}{Avg2\cdot \mathrm{LN}\left(P^{-\,(Avg1\cdot\mu t + \sqrt{(Avg2^2 + k2)} + Vct\cdot\mu t)/\sqrt{(Avg2^2 + k2)}}\cdot\left(P^{(\mu t\cdot(Aik\cdot Gk))^{1/n1}/Avg2} - 1\right)\right)}\right)\right)$$

Simplifying (26):

#27:

$$\mu p = \frac{\mu t\cdot(Avp - Avg2)\cdot \mathrm{LN}(P)}{Avg2\cdot \mathrm{LN}\left(P^{-\,(Avg1\cdot\mu t + \sqrt{(Avg2^2 + k2)} + Vct\cdot\mu t)/\sqrt{(Avg2^2 + k2)}}\cdot\left(P^{(\mu t\cdot(Aik\cdot Gk))^{1/n1}/Avg2} -1\right)\right)}$$

Solving (20) for μp:

$$\#28: \quad \mu p = \frac{\mu g2 \cdot \mu t}{\mu g2 - \mu t}$$

Substituting μp back into (22) and then replacing vg2 with vp and simplifying, reduces (22) to (15), the triode cathode current equation, as shown in (29):

$$\#29: \quad ik = \frac{1}{Gk} \cdot \left(\frac{vp}{\mu t} \cdot \frac{LN\left(P^{(vg1 + Vct) \cdot \mu t / \sqrt{(k2 + vp^2)} + 1} + 1\right)}{LN(P)} \right)^{n1} \cdot \frac{1 + SIGN(vp)}{2}$$

Calculation of the triode connection parameters n1, μt, k2, P and Gk:

Substituting constants for the variables in (15):

$$\#30: \quad B1ik = \frac{1}{Gk} \cdot \left(\frac{B1vp}{\mu t} \cdot \frac{LN\left(P^{(B1vg1 + Vct) \cdot \mu t / \sqrt{(k2 + B1vp^2)} + 1} + 1\right)}{LN(P)} \right)^{n1} \cdot \frac{1 + SIGN(B1vp)}{2}$$

Solving (30) for Gk:

$$\#31: \quad Gk = \frac{1}{B1ik} \cdot \left(\frac{B1vp}{\mu t} \cdot \frac{LN\left(P^{(B1vg1 + Vct) \cdot \mu t / \sqrt{(k2 + B1vp^2)} + 1} + 1\right)}{LN(P)} \right)^{n1} \cdot \frac{1 + SIGN(B1vp)}{2}$$

Substituting zero for (B1vg1 + Vct) in (31):

$$\#32: \quad Gk = \frac{1}{B1ik} \cdot \left(\frac{B1vp}{\mu t} \cdot \frac{LN\left(P^{0 \cdot \mu t / \sqrt{(k2 + B1vp^2)} + 1} + 1\right)}{LN(P)} \right)^{n1} \cdot \frac{1 + SIGN(B1vp)}{2}$$

Simplifying (32):

$$\#33: \quad Gk = \frac{1}{B1ik} \cdot \left(\frac{B1vp}{\mu t} \cdot \frac{LN(P + 1)}{LN(P)} \right)^{n1} \cdot \frac{1 + SIGN(B1vp)}{2}$$

Substituting a new set of constants for the variables in (15):

#34:
$$B2ik = \frac{1}{Gk}\cdot\left(\frac{B2vp}{\mu t}\cdot\frac{LN\left(P^{(B2vg1 + Vct)\cdot\mu t/\sqrt{(k2 + B2vp^2)} + 1} + 1\right)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(B2vp)}{2}$$

Substituting zero for (B2vg1 + Vct) in (34):

#35:
$$B2ik = \frac{1}{Gk}\cdot\left(\frac{B2vp}{\mu t}\cdot\frac{LN\left(P^{0\cdot\mu t/\sqrt{(k2 + B2vp^2)} + 1} + 1\right)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(B2vp)}{2}$$

Simplifying (35):

#36:
$$B2ik = \frac{1}{Gk}\cdot\left(\frac{B2vp}{\mu t}\cdot\frac{LN(P + 1)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(B2vp)}{2}$$

Substituting for Gk in (36):

#37:

$$B2ik = \frac{1}{\frac{1}{B1ik}\cdot\left(\frac{B1vp}{\mu t}\cdot\frac{LN(P + 1)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(B1vp)}{2}}\cdot\left(\frac{B2vp}{\mu t}\cdot\frac{LN(P + 1)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(B2vp)}{2}$$

Simplifying (37):

#38:
$$B2ik = \frac{B1ik}{B1vp^{n1}}\cdot B2vp^{n1}\cdot\frac{1 + SIGN(B2vp)}{1 + SIGN(B1vp)}$$

Solving (38) for n1:

#39:
$$n1 = \frac{LN\left(\frac{B2ik\cdot(SIGN(B1vp) + 1)}{B1ik\cdot(SIGN(B2vp) + 1)}\right)}{LN(B2vp) - LN(B1vp)}$$

Entering the zero bias line data for the triode connected 6550 example:

#40: $[B1ik := 0.502,\ B1vp := 300,\ B2ik := 0.176,\ B2vp := 150]$

Simplifying (39) and assigning the resultant value to n1:

#41: $n1 := 1.512111935$

Substituting for Gk in (15):

#42: $ik =$

$$\frac{1}{\frac{1}{B1ik}\cdot\left(\frac{B1vp}{\mu t}\cdot\frac{LN(P + 1)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(B1vp)}{2}}\cdot\left(\frac{vp}{\mu t}\cdot\frac{LN\left(P^{(vg1 + Vct)\cdot\mu t/\sqrt{(k2 + vp^{2})} + 1} + 1\right)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Simplifying (42):

#43: $$ik = \frac{B1ik}{B1vp^{n1}}\cdot\left(vp\cdot\frac{LN\left(P^{(vg1 + Vct)\cdot\mu t/\sqrt{(k2 + vp^{2})} + 1} + 1\right)}{LN(P + 1)}\right)^{n1}\cdot\frac{1 + SIGN(vp)}{1 + SIGN(B1vp)}$$

Substituting constants for the variables in (43):

#44: $$C1ik = \frac{B1ik}{B1vp^{n1}}\cdot\left(C1vp\cdot\frac{LN\left(P^{(C1vg1 + Vct)\cdot\mu t/\sqrt{(k2 + C1vp^{2})} + 1} + 1\right)}{LN(P + 1)}\right)^{n1}\cdot\frac{1 + SIGN(C1vp)}{1 + SIGN(B1vp)}$$

Entering the data for C1ik, C1vp, C1vg1 and Vct:

#45: [C1ik := 0.005, C1vp := 80, C1vg1 := -10, Vct := 0]

Simplifying (44):

#46: $$\frac{\mathrm{LN}\left(P^{-10\cdot\mu t/\sqrt{(k2 + 6400)}}\cdot\left(P^{10\cdot\mu t/\sqrt{(k2 + 6400)}} + P\right)\right)}{\mathrm{LN}(P + 1)} = 0.1779226801$$

Solving (46) for μt and assigning it's value to μt:

#47: $$\mu t := \frac{\sqrt{(k2 + 6400)}\cdot\mathrm{LN}\left(\dfrac{P}{(P + 1)^{1779226801/10000000000} - 1}\right)}{10\cdot\mathrm{LN}(P)}$$

Substituting a new set of constants for the variables in (43):

#48: $$C2ik = \frac{B1ik}{B1vp^{n1}}\cdot\left(C2vp\cdot\frac{\mathrm{LN}\left(P^{(C2vg1 + Vct)\cdot\mu t/\sqrt{(k2 + C2vp^2)} + 1} + 1\right)}{\mathrm{LN}(P + 1)}\right)^{n1}\cdot\frac{1 + \mathrm{SIGN}(C2vp)}{1 + \mathrm{SIGN}(B1vp)}$$

Entering the data for C2ik, C2vp, C2vg1 and Vct:

#49: [C2ik := 0.035, C2vp := 520, C2vg1 := -70, Vct := 0]

Simplifying (48):

#50: $$\frac{\mathrm{LN}\left(\left(\left(\dfrac{P}{(P + 1)^{0.17792268} - 1}\right)^{7\cdot\sqrt{(k2 + 6400)}/\sqrt{(k2 + 2.704\cdot 10^5)}} + P\right)\cdot\left(\dfrac{P}{(P + 1)^{0.17792268} - 1}\right)^{-7\cdot\sqrt{(k2 + 6400)}/\sqrt{(k2 + 2.704\cdot 10^5)}}\right)}{\mathrm{LN}(P + 1)} = 0.09912971451$$

Solving (50) for k2 and assigning it's value to k2:

#51: k2 :=

$$\frac{1600\cdot\left(196\cdot LN\left(\frac{P}{(P+1)^{4448067/25000000}-1}\right)^2-169\cdot LN\left(\frac{P}{(P+1)^{9912971451/100000000000}-1}\right)^2\right)}{LN\left(\frac{P}{(P+1)^{9912971451/100000000000}-1}\right)^2-49\cdot LN\left(\frac{P}{(P+1)^{4448067/25000000}-1}\right)^2}$$

Substituting another new set of constants for the variables in (43):

$$\#52:\quad Dik = \frac{B1ik}{B1vp^{n1}}\cdot\left(Dvp\cdot\frac{LN\left(P^{(Dvg1+Vct)\cdot\mu t/\sqrt{(k2+Dvp^2)}+1}+1\right)}{LN(P+1)}\right)^{n1}\cdot\frac{1+SIGN(Dvp)}{1+SIGN(B1vp)}$$

Entering the data for Dik, Dvp and Dvg1 and Vct:

#53: [Dik := 0.151, Dvp := 300, Dvg1 := -20, Vct := 0]

Solving (52) and assigning it's value to P:

#54: P := 39.10442113

Simplifying (51):

#55: k2 := 1573.570786

Simplifying (47):

#56: μt := 9.109883328

Simplifying (33) and assigning it's value to Gk:

#57: Gk := 396.8237161

PENTODE CONNECTION CATHODE CURRENT:

Calculation of the pentode connection parameters μp, a, b and n2:

Entering the data for Aik, Avp, Avg2 and Avg1:

#58: [Aik ≔ 0.138, Avp ≔ 80, Avg2 ≔ 300, Avg1 ≔ −20]

Simplifying (27) and assigning it's value to μp:

#59: μp ≔ 203.7413383

PENTODE CONNECTION PLATE CURRENT:

Pentode knee equation:

#60:
$$ip = \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp} \cdot ik$$

Substituting for ik in (60):

#61: ip =

$$\frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp} \cdot \frac{1}{Gk} \cdot \left(\frac{vg2}{\mu t} \cdot \frac{LN\left(P^{(vg1 + Vct)\cdot\mu t/\sqrt{(k2 + vg2^2)} + vp\cdot\mu t/(vg2\cdot\mu p) + (\mu p - \mu t)/\mu p} + 1\right)}{LN(P)}\right)^{n1} \cdot \frac{1 + SIGN(vp)}{2}$$

Substituting constants for the variables in (61):

#62: E1ip =

$$\frac{a + E1vp}{\left(\frac{Evg2}{E1vp}\right)^{n2} + b + E1vp} \cdot \frac{1}{Gk} \cdot \left(\frac{Evg2}{\mu t} \cdot \frac{LN\left(P^{(E1vg1 + Vct)\cdot\mu t/\sqrt{(k2 + Evg2^2)} + E1vp\cdot\mu t/(Evg2\cdot\mu p) + (\mu p - \mu t)/\mu p} + 1\right)}{LN(P)}\right)^{n1} \cdot \frac{1 + SIGN(E1vp)}{2}$$

Entering the data for Evg2, E1ip, E1vp and E1vg1:

#63: $[Evg2 := 300,\ E1ip := 0.452,\ E1vp := 300,\ E1vg1 := 0,\ Vct := 0]$

Simplifying (62):

#64: $$\frac{a + 300}{b + 301} = 0.9003984063$$

Solving (64) for a and assigning it's value to a:

#65: $$a := \frac{3\cdot(3001328021\cdot b - 96600265679)}{10000000000}$$

Substituting a new set of constants for the variables in (61):

#66: E2ip =

$$\frac{a + E2vp}{\left(\frac{Evg2}{E2vp}\right)^{n2} + b + E2vp}\cdot\frac{1}{Gk}\cdot\left(\frac{Evg2}{\mu t}\cdot\frac{LN\left(P^{(E2vg1 + Vct)\cdot\mu t/\sqrt{(k2 + Evg2^2)} + E2vp\cdot\mu t/(Evg2\cdot\mu p) + (\mu p - \mu t)/\mu p} + 1\right)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(E2vp)}{2}$$

Entering the data for E2ip, E2vp, E2vg1 and Vct:

#67: $[E2ip := 0.425,\ E2vp := 80,\ E2vg1 := 0,\ Vct := 0]$

Simplifying (66):

#68: $$\frac{9.003984063\cdot 10^{9}\cdot b + 5.101992029\cdot 10^{11}}{3.75^{n2} + b + 80} = 8.888783259\cdot 10^{9}$$

Solving (68) for b and assigning it's value to b:

#69: $$b := \frac{2962927753\cdot 2^{-2\cdot(n2 + 1)}\cdot 15^{n2}}{9600067} + \frac{50225864455}{28800201}$$

Substituting another new set of constants for the variables in (61):

#70: E3ip =

$$\frac{a + E3vp}{\left(\frac{Evg2}{E3vp}\right)^{n2} + b + E3vp}\cdot\frac{1}{Gk}\cdot\left(\frac{Evg2}{\mu t}\cdot\frac{LN\left(P^{(E3vg1 + Vct)\cdot\mu t/\sqrt{(k2 + Evg2^{2})} + E3vp\cdot\mu t/(Evg2\cdot\mu p) + (\mu p - \mu t)/\mu p} + 1\right)}{LN(P)}\right)^{n1}\cdot\frac{1 + SIGN(E3vp)}{2}$$

Entering the data for E3ip, E3vp, E3vg1 and Vct:

#71: [E3ip := 0.15, E3vp := 20, E3vg1 := 0, Vct := 0]

Simplifying (70):

#72: $$\frac{3.425495798\cdot 10^{10}\cdot 3.75^{n2} + 7.69799655\cdot 10^{11}}{1.582236235\cdot 10^{10}\cdot 15^{n2} + 1.220838264\cdot 10^{12}\cdot 3.75^{n2} + 2.790972088\cdot 10^{13}} = 0.00990953597$$

Solving (72) for n2 and assigning its value to n2:

#73: n2 := 3.685059454

Simplifying (69):

#74: $b := 1.180691096\cdot 10^{4}$

Simplifying (65):

#75: $a := 1.060194373\cdot 10^{4}$

DERIVE 6 PENTODE PARAMETER PROGRAM (6550 EXAMPLE)

```
      PENTODE_Prog(P) :=
        Prog
          [Vp :=, Vt :=, Vct :=, Aik :=, Avp :=, Avg2 :=, Avg1 :=]
          [B1ik :=, B1vp :=, B1vg1 :=, B2ik :=, B2vp :=, B2vg1 :=]
 CLEAR    [C1ik :=, C1vp :=, C1vg1 :=, C2ik :=, C2vp :=, C2vg1 :=]
 DATA     [Dik :=, Dvp :=, Dvg2 :=, Dvg1 :=]
          [Evg2 :=, E1ip :=, E1vp :=, E1vg1 :=, E2ip :=, E2vp :=, E2vg1 :=, E3ip :=, E3vp :=, E3vg1 :=]
          [n1 :=, μt :=, μg2 :=, μp :=, k2 :=, P :=, Gk :=, a :=, b :=, n2 :=]


          Aik := 0.138
          Avp := 80
          Avg2 := 300
          Avg1 := -20
          B1ik := 0.502
          B1vp := 300
          B1vg1 := 0
          B2ik := 0.176
          B2vp := 150
          B2vg1 := 0
          C1ik := 0.005
          C1vp := 80
          C1vg1 := -10
          Vct := 0
 ENTER    C2ik := 0.035
 DATA     C2vp := 520
          C2vg1 := -70
          Dik := 0.151
          Dvp := 300
#76:      Dvg1 := -20
          Evg2 := 300
          E1ip := 0.452
          E1vp := 300
          E1vg1 := 0
          E2ip := 0.425
          E2vp := 80
          E2vg1 := 0
          E3ip := 0.15
          E3vp := 20
          E3vg1 := 0
```

```
        n1 ≔ LN(B2ik·(SIGN(B1vp) + 1)/(B1ik·(SIGN(B2vp) + 1)))/(LN(B2vp) − LN(B1vp))
        ASSIGN(μt, SOLVE(C1ik = B1ik/B1vp^n1·(C1vp·(LN(P^((C1vg1 + Vct)·μt/√(k2 + C1vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C1vp))/(1 + SIGN(B1vp))),
                      μt, Real))
        μt ≔ RHS(μt)
        ASSIGN(k2, SOLVE(C2ik = B1ik/B1vp^n1·(C2vp·(LN(P^((C2vg1 + Vct)·μt/√(k2 + C2vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C2vp))/(1 + SIGN(B1vp))),
                      k2, Real))
        k2 ≔ RHS(k2)
        ASSIGN(P, NSOLVE(Dik = B1ik/B1vp^n1·(Dvp·(LN(P^((Dvg1 + Vct)·μt/√(k2 + Dvp^2) + 1)+ 1)/LN(P + 1)))^n1·((1 + SIGN(Dvp))/(1 + SIGN(B1vp))), P, Real))
PROGRAM P ≔ RHS(P)
        Gk ≔ 1/B1ik·(B1vp/μt·(LN(P + 1)/LN(P)))^n1·((1 + SIGN(B1vp))/2)
        μp ≔ μt·(Avp − Avg2)·LN(P)/(Avg2·LN(P^(− (Avg1·μt + √(Avg2^2 + k2) + Vct·μt)/√(Avg2^2 + k2))·(P^(μt·(Aik·Gk)^(1/n1)/Avg2) − 1)))
        ASSIGN(a, SOLVE(E1ip = (a + E1vp)/((Evg2/E1vp)^n2 + b + E1vp)·(1/Gk)·(Evg2/μt·(LN(P^((E1vg1 + Vct)·μt/√(k2 + Evg2^2) + E1vp·μt/(Evg2·μp) +
                      (μp − μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E1vp))/2), a, Real))
        a ≔ RHS(a)
        ASSIGN(b, SOLVE(E2ip = (a + E2vp)/((Evg2/E2vp)^n2 + b + E2vp)·(1/Gk)·(Evg2/μt·(LN(P^((E2vg1 + Vct)·μt/√(k2 + Evg2^2) + E2vp·μt/(Evg2·μp) +
                      (μp − μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E2vp))/2), b, Real))
        b ≔ RHS(b)
        ASSIGN(n2, NSOLVE(E3ip = (a + E3vp)/((Evg2/E3vp)^n2 + b + E3vp)·(1/Gk)·(Evg2/μt·(LN(P^((E3vg1 + Vct)·μt/√(k2 + Evg2^2) + E3vp·μt/(Evg2·μp) +
                      (μp − μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E3vp))/2), n2, Real))
        n2 ≔ RHS(n2)
        [n1 ≔ n1, μt ≔ μt, k2 ≔ k2, P ≔ P, Gk ≔ Gk, μp ≔ μp, a ≔ a, b ≔ b, n2 ≔ n2]
```

6550 Pentode Solutions:

```
#77:  PENTODE_Prog(P) ≔ [n1 ≔ 1.512111935, μt ≔ 9.10988332, k2 ≔ 1573.57079, P ≔ 39.10442143,
                                                           4                       4
        Gk ≔ 396.8237171, μp ≔ 203.7413381, a ≔ 1.060194404·10 , b ≔ 1.18069113·10 , n2 ≔  3.685059472]
```

DERIVE 6 PENTODE PARAMETER PROGRAM (EL34 EXAMPLE)

```
     PENTODE_Prog(P) :=
       Prog
         [Vp :=, Vt :=, Vct :=, Aik :=, Avp :=, Avg2 :=, Avg1 :=]
 CLEAR   [B1ik :=, B1vp :=, B1vg1 :=, B2ik :=, B2vp :=, B2vg1 :=]
 DATA    [C1ik :=, C1vp :=, C1vg1 :=, C2ik :=, C2vp :=, C2vg1 :=]
         [Dik :=, Dvp :=, Dvg2 :=, Dvg1 :=]
         [Evg2 :=, E1ip :=, E1vp :=, E1vg1 :=, E2ip :=, E2vp :=, E2vg1 :=, E3ip :=, E3vp :=, E3vg1 :=]
         [n1 :=, μt :=, μg2 :=, μp :=, k2 :=, P :=, Gk :=, a :=, b :=, n2 :=]


         Aik := 0.06
         Avp := 140
         Avg2 := 250
         Avg1 := -15
         B1ik := 0.255
         B1vp := 210
         B1vg1 := 0
         B2ik := 0.086
         B2vp := 100
         B2vg1 := 0
 ENTER   C1ik := 0.004
 DATA    C1vp := 30
         C1vg1 := -5
         Vct := 0
         C2ik := 0.005
         C2vp := 290
         C2vg1 := -35
         Dik := 0.034
         Dvp := 250
#78:     Dvg1 := -20
         Evg2 := 250
         E1ip := 0.281
         E1vp := 250
         E1vg1 := 0
         E2ip := 0.22
         E2vp := 65
         E2vg1 := 0
         E3ip := 0.1
         E3vp := 14
         E3vg1 := 0
```

```
          n1 ≔ LN(B2ik·(SIGN(B1vp) + 1)/(B1ik·(SIGN(B2vp) + 1)))/(LN(B2vp) - LN(B1vp))
             ASSIGN(μt, SOLVE(C1ik = B1ik/B1vp^n1·(C1vp·(LN(P^((C1vg1 + Vct)·μt/√(k2 + C1vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C1vp))/(1 + SIGN(B1vp))),
                              μt, Real))
             μt ≔ RHS(μt)
             ASSIGN(k2, SOLVE(C2ik = B1ik/B1vp^n1·(C2vp·(LN(P^((C2vg1 + Vct)·μt/√(k2 + C2vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C2vp))/(1 + SIGN(B1vp))),
                              k2, Real))
             k2 ≔ RHS(k2)
             ASSIGN(P, NSOLVE(Dik = B1ik/B1vp^n1·(Dvp·(LN(P^((Dvg1 + Vct)·μt/√(k2 + Dvp^2) + 1)+ 1)/LN(P + 1)))^n1·((1 + SIGN(Dvp))/(1 + SIGN(B1vp))), P, Real))
             P ≔ RHS(P)
             Gk ≔ 1/B1ik·(B1vp/μt·(LN(P + 1)/LN(P)))^n1·((1 + SIGN(B1vp))/2)
PROGRAM      μp ≔ μt·(Avp - Avg2)·LN(P)/(Avg2·LN(P^(- (Avg1·μt + √(Avg2^2 + k2) + Vct·μt)/√(Avg2^2 + k2))·(P^(μt·(Aik·Gk)^(1/n1)/Avg2) - 1)))
             ASSIGN(a, SOLVE(E1ip = (a + E1vp)/((Evg2/E1vp)^n2 + b + E1vp)·(1/Gk)·(Evg2/μt·(LN(P^((E1vg1 + Vct)·μt/√(k2 + Evg2^2) + E1vp·μt/(Evg2·μp) +
                              (μp - μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E1vp))/2), a, Real))
             a ≔ RHS(a)
             ASSIGN(b, SOLVE(E2ip = (a + E2vp)/((Evg2/E2vp)^n2 + b + E2vp)·(1/Gk)·(Evg2/μt·(LN(P^((E2vg1 + Vct)·μt/√(k2 + Evg2^2) + E2vp·μt/(Evg2·μp) +
                              (μp - μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E2vp))/2), b, Real))
             b ≔ RHS(b)
             ASSIGN(n2, NSOLVE(E3ip = (a + E3vp)/((Evg2/E3vp)^n2 + b + E3vp)·(1/Gk)·(Evg2/μt·(LN(P^((E3vg1 + Vct)·μt/√(k2 + Evg2^2) + E3vp·μt/(Evg2·μp) +
                              (μp - μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E3vp))/2), n2, Real))
             n2 ≔ RHS(n2)
             [n1 ≔ n1, μt ≔ μt, k2 ≔ k2, P ≔ P, Gk ≔ Gk, μp ≔ μp, a ≔ a, b ≔ b, n2 ≔ n2]
```

EL34 Pentode Solutions:

```
#79:  PENTODE_Prog(P) ≔ [n1 ≔ 1.4649704, μt ≔ 13.18149295, k2 ≔ 8710.35021, P ≔ 27.80139923,

        Gk ≔ 229.854627, μp ≔ 80.33273163, a ≔ 619.2087066, b ≔ 767.3252981, n2 ≔ 2.348941862]
```

DERIVE 6 PENTODE PARAMETER PROGRAM (6BQ5 EXAMPLE)

```
      PENTODE_Prog(P) :=
        Prog
          [Vp :=, Vt :=, Vct :=, Aik :=, Avp :=, Avg2 :=, Avg1 :=]
          [B1ik :=, B1vp :=, B1vg1 :=, B2ik :=, B2vp :=, B2vg1 :=]
 CLEAR    [C1ik :=, C1vp :=, C1vg1 :=, C2ik :=, C2vp :=, C2vg1 :=]
 DATA     [Dik :=, Dvp :=, Dvg2 :=, Dvg1 :=]
          [Evg2 :=, E1ip :=, E1vp :=, E1vg1 :=, E2ip :=, E2vp :=, E2vg1 :=, E3ip :=, E3vp :=, E3vg1 :=]
          [n1 :=, μt :=, μg2 :=, μp :=, k2 :=, P :=, Gk :=, a :=, b :=, n2 :=]


          Aik := 0.023
          Avp := 80
          Avg2 := 250
          Avg1 := -10
          B1ik := 0.35
          B1vp := 395
          B1vg1 := 0
          B2ik := 0.135
          B2vp := 200
          B2vg1 := 0
          C1ik := 0.005
          C1vp := 47
          C1vg1 := -2
          Vct := 0
 ENTER    C2ik := 0.005
 DATA     C2vp := 337.5
          C2vg1 := -20
          Dik := 0.025
          Dvp := 250
#80:      Dvg1 := -10
          Evg2 := 250
          E1ip := 0.161
          E1vp := 250
          E1vg1 := 0
          E2ip := 0.144
          E2vp := 80
          E2vg1 := 0
          E3ip := 0.07
          E3vp := 20
          E3vg1 := 0
```

```
         n1 ≔ LN(B2ik·(SIGN(B1vp) + 1)/(B1ik·(SIGN(B2vp) + 1)))/(LN(B2vp) - LN(B1vp))
         ASSIGN(μt, SOLVE(C1ik = B1ik/B1vp^n1·(C1vp·(LN(P^((C1vg1 + Vct)·μt/√(k2 + C1vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C1vp))/(1 + SIGN(B1vp))),
                        μt, Real))
         μt ≔ RHS(μt)
         ASSIGN(k2, SOLVE(C2ik = B1ik/B1vp^n1·(C2vp·(LN(P^((C2vg1 + Vct)·μt/√(k2 + C2vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C2vp))/(1 + SIGN(B1vp))),
                        k2, Real))
         k2 ≔ RHS(k2)
         ASSIGN(P, NSOLVE(Dik = B1ik/B1vp^n1·(Dvp·(LN(P^((Dvg1 + Vct)·μt/√(k2 + Dvp^2) + 1)+ 1)/LN(P + 1)))^n1·((1 + SIGN(Dvp))/(1 + SIGN(B1vp))), P, Real))
         P ≔ RHS(P)
 PROGRAM  Gk ≔ 1/B1ik·(B1vp/μt·(LN(P + 1)/LN(P)))^n1·((1 + SIGN(B1vp))/2)
         μp ≔ μt·(Avp - Avg2)·LN(P)/(Avg2·LN(P^(- (Avg1·μt + √(Avg2^2 + k2) + Vct·μt)/√(Avg2^2 + k2))·(P^(μt·(Aik·Gk)^(1/n1)/Avg2) - 1)))
         ASSIGN(a, SOLVE(E1ip = (a + E1vp)/((Evg2/E1vp)^n2 + b + E1vp)·(1/Gk)·(Evg2/μt·(LN(P^((E1vg1 + Vct)·μt/√(k2 + Evg2^2) + E1vp·μt/(Evg2·μp) +
                        (μp - μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E1vp))/2), a, Real))
         a ≔ RHS(a)
         ASSIGN(b, SOLVE(E2ip = (a + E2vp)/((Evg2/E2vp)^n2 + b + E2vp)·(1/Gk)·(Evg2/μt·(LN(P^((E2vg1 + Vct)·μt/√(k2 + Evg2^2) + E2vp·μt/(Evg2·μp) +
                        (μp - μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E2vp))/2), b, Real))
         b ≔ RHS(b)
         ASSIGN(n2, NSOLVE(E3ip = (a + E3vp)/((Evg2/E3vp)^n2 + b + E3vp)·(1/Gk)·(Evg2/μt·(LN(P^((E3vg1 + Vct)·μt/√(k2 + Evg2^2) + E3vp·μt/(Evg2·μp) +
                        (μp - μt)/μp) + 1)/LN(P)))^n1·((1 + SIGN(E3vp))/2), n2, Real))
         n2 ≔ RHS(n2)
         [n1 ≔ n1, μt ≔ μt, k2 ≔ k2, P ≔ P, Gk ≔ Gk, μp ≔ μp, a ≔ a, b ≔ b, n2 ≔ n2]
```

EL84/6BQ5 Pentode Solutions:

```
#81:  PENTODE_Prog(P) ≔ [n1 ≔ 1.399798136, μt ≔ 21.62052219, k2 ≔ 2465.831633, P ≔ 96.84356689,

        Gk ≔ 167.2879461, μp ≔ 693.8667584, a ≔ 322.5621, b ≔ 405.1225215, n2 ≔ 2.411887627]
```

PENTODE MODEL EQUATIONS
TRIODE CONNECTION EQUATION:

#82: $$ik = \frac{1}{Gk} \cdot Vt^{n1} \cdot \frac{1 + SIGN(vp)}{2}$$

Where:

#83: $$Vt^{n1} = \sqrt{\left(\left(\left|\frac{Vt}{2}\right| + \frac{Vt}{2}\right) \cdot Vt^{2 \cdot n1 - 1}\right)}$$

Substituting (83) into (82):

#84: $$ik = \frac{1}{Gk} \cdot \sqrt{\left(\left(\left|\frac{Vt}{2}\right| + \frac{Vt}{2}\right) \cdot Vt^{2 \cdot n1 - 1}\right)} \cdot \frac{1 + SIGN(vp)}{2}$$

And:

#85: $$Vt := \frac{vp}{\mu t} \cdot \frac{LN\left(P^{(vg1 + Vct) \cdot \mu t/\sqrt{(k2 + vp^2)} + 1} + 1\right)}{LN(P)}$$

PENTODE CONNECTION EQUATIONS:

Cathode Current:

#86: $$ik = \frac{1}{Gk} \cdot Vp^{n1} \cdot \frac{1 + SIGN(vp)}{2}$$

Where:

#87: $$Vp^{n1} = \sqrt{\left(\left(\left|\frac{Vp}{2}\right| + \frac{Vp}{2}\right) \cdot Vp^{2 \cdot n1 - 1}\right)}$$

Substituting (87) into (86):

#88: $$ik = \frac{1}{Gk} \cdot \sqrt{\left(\left(\left|\frac{Vp}{2}\right| + \frac{Vp}{2}\right) \cdot Vp^{2 \cdot n1 - 1}\right)} \cdot \frac{1 + SIGN(vp)}{2}$$

Plate Current:

#89: $$ip = \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp}\cdot ik$$

Substituting for ik in (89):

#90: $$ip = \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp}\cdot\frac{1}{Gk}\cdot Vp^{n1}\cdot\frac{1 + SIGN(vp)}{2}$$

Where:

#91: $$Vp^{n1} = \sqrt{\left(\left(\left|\frac{Vp}{2}\right| + \frac{Vp}{2}\right)\cdot Vp^{2\cdot n1 - 1}\right)}$$

Substituting (91) into (90):

#92: $$ip = \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp}\cdot\frac{1}{Gk}\cdot\sqrt{\left(\left(\left|\frac{Vp}{2}\right| + \frac{Vp}{2}\right)\cdot Vp^{2\cdot n1 - 1}\right)}\cdot\frac{1 + SIGN(vp)}{2}$$

Screen Grid Current:

#93: $ig2 = ik - ip$

Substituting for ip in (93):

#94: $$ig2 = ik - \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp}\cdot ik$$

Factoring out ik in (94):

#95: $$ig2 = \left(1 - \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp}\right)\cdot ik$$

Substituting for ik in (95):

#96: $$ig2 = \left(1 - \frac{a + vp}{\left(\frac{vg2}{vp}\right)^{n2} + b + vp}\right)\cdot\left(\frac{1}{Gk}\cdot\sqrt{\left(\left(\left|\frac{Vp}{2}\right| + \frac{Vp}{2}\right)\cdot Vp^{2\cdot n1 - 1}\right)}\cdot\frac{1 + SIGN(vp)}{2}\right)$$

Where:

#97: $$Vp := \frac{vg2}{\mu t}\cdot\frac{LN\left(P^{(vg1 + Vct)\cdot\mu t/\sqrt{(k2 + vg2^{2})} + vp\cdot\mu t/(vg2\cdot\mu p) + (\mu p - \mu t)/\mu p} + 1\right)}{LN(P)}$$

A value for vg2 must be supplied to graph the pentode equation with the Derive 6 plotter. Otherwise it is not required:

#98: $vg2 :=$

Data Point Selection:

The most crucial data point is the one I call C1. For low, medium and lower high mu triodes (e.g. 12AT7/12AZ7), It MUST be located on the first negative control grid bias line near cutoff . On very high mu triodes (e.g. 12AX7), it MUST be located on the second negative control grid bias line near cutoff and the "contact potential" Vct must be taken into account. The selection of the other data points B1, B2, C2 and D are important, but not nearly so much as C1. (For very high mu triodes, B1 and B2 MUST be located on first negative grid bias line).

There are 5 data points to be placed on the triode plate charateristics. A1 and A2 are placed on the zero control grid bias line (Except for very high μ triodes). A1 is placed near the highest plate current of interest. A2 should be placed at about half the plate voltage of A1. B1 is placed on the first negative grid bias line near cutoff (Except for very high μ triodes). B2 is placed on the most negative grid bias line of interest near cutoff. C is placed somewhere in the center of the plate characteristics.

DERIVE 6 TRIODE PARAMETER PROGRAM (12AU7 EXAMPLE)
Equations (44), (48) and (52) are used in the triode program to calculate the values of μ, k2 and P.

```
      TRIODE_Prog(P) :=
        Prog
          [Vt :=, Vct :=]
 CLEAR    [B1ik :=, B1vp :=, B1vg1 :=, B2ik :=, B2vp :=, B2vg1 :=]
 DATA     [C1ik :=, C1vp :=, C1vg1 :=, C2ik :=, C2vp :=, C2vg1 :=]
          [Dik :=, Dvp :=, Dvg2 :=, Dvg1 :=]
          [n1 :=, μt :=, k2 :=, P :=, Gk :=]

          Vct := 0
          B1ik := 0.036
          B1vp := 225
          B1vg1 := 0
          B2ik := 0.016
          B2vp := 125
 ENTER    B2vg1 := 0
 DATA     C1ik := 0.0005
          C1vp := 40
#99:      C1vg1 := -2
          C2ik := 0.0005
          C2vp := 420
          C2vg1 := -30
          Dik := 0.0115
          Dvp := 250
          Dvg1 := -8
```

```
          n1 ≔ LN(B2ik·(SIGN(B1vp) + 1)/(B1ik·(SIGN(B2vp) + 1)))/(LN(B2vp) − LN(B1vp))
          ASSIGN(μt, SOLVE(C1ik = B1ik/B1vp^n1·(C1vp·(LN(P^((C1vg1 + Vct)·μt/√(k2 + C1vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C1vp))/(1 + SIGN(B1vp))),
                         μt, Real))
          μt ≔ RHS(μt)
 PROGRAM  ASSIGN(k2, SOLVE(C2ik = B1ik/B1vp^n1·(C2vp·(LN(P^((C2vg1 + Vct)·μt/√(k2 + C2vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C2vp))/(1 + SIGN(B1vp))),
                         k2, Real))
          k2 ≔ RHS(k2)
          ASSIGN(P, NSOLVE(Dik = B1ik/B1vp^n1·(Dvp·(LN(P^((Dvg1 + Vct)·μt/√(k2 + Dvp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(Dvp))/(1 + SIGN(B1vp))), P, Real))
          P ≔ RHS(P)
          Gk ≔ 1/B1ik·(B1vp/μt·(LN(P + 1)/LN(P)))^n1·((1 + SIGN(B1vp))/2)
          [n1 ≔ n1, μt ≔ μt, k2 ≔ k2, P ≔ P, Gk ≔ Gk]
```

12AU7 Triode Solutions:

```
#100: TRIODE_Prog(P) ≔ [n1 ≔ 1.379633572, μt ≔ 20.1067084, k2 ≔ 815.6995298, P ≔ 124.7750759, Gk ≔ 779.2999176]
```

Data points B1 and B2 are placed on the first negative grid bias line for very high μ triodes such as the 12AX7. Data point C1 is placed on the second negative grid bias line near cutoff. Data points C2 and D are placed the same as for lower μ triodes.

DERIVE 6 TRIODE PARAMETER PROGRAM (12AX7 EXAMPLE)

```
      TRIODE_Prog(P) ≔
        Prog
          [Vt ≔, Vct ≔]
 CLEAR    [B1ik ≔, B1vp ≔, B1vg1 ≔, B2ik ≔, B2vp ≔, B2vg1 ≔]
 DATA     [C1ik ≔, C1vp ≔, C1vg1 ≔, C2ik ≔, C2vp ≔, C2vg1 ≔]
          [Dik ≔, Dvp ≔, Dvg2 ≔, Dvg1 ≔]
          [n1 ≔, μt ≔, k2 ≔, P ≔, Gk ≔]

          Vct ≔ 0.5
          B1ik ≔ 0.00325
          B1vp ≔ 200
          B1vg1 ≔ −0.5
          B2ik ≔ 0.00025
          B2vp ≔ 30
 ENTER    B2vg1 ≔ −0.5
 DATA     C1ik ≔ 0.0001
          C1vp ≔ 50
#101:     C1vg1 ≔ −1
          C2ik ≔ 0.0001
          C2vp ≔ 400
          C2vg1 ≔ −5
          Dik ≔ 0.0012
          Dvp ≔ 200
          Dvg1 ≔ −1.5
```

```
         n1 := LN(B2ik·(SIGN(B1vp) + 1)/(B1ik·(SIGN(B2vp) + 1)))/(LN(B2vp) - LN(B1vp))
         ASSIGN(μt, SOLVE(C1ik = B1ik/B1vp^n1·(C1vp·(LN(P^((C1vg1 + Vct)·μt/√(k2 + C1vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C1vp))/(1 + SIGN(B1vp))),
                        μt, Real))
         μt := RHS(μt)
 PROGRAM ASSIGN(k2, SOLVE(C2ik = B1ik/B1vp^n1·(C2vp·(LN(P^((C2vg1 + Vct)·μt/√(k2 + C2vp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(C2vp))/(1 + SIGN(B1vp))),
                        k2, Real))
         k2 := RHS(k2)
         ASSIGN(P, NSOLVE(Dik = B1ik/B1vp^n1·(Dvp·(LN(P^((Dvg1 + Vct)·μt/√(k2 + Dvp^2) + 1) + 1)/LN(P + 1)))^n1·((1 + SIGN(Dvp))/(1 + SIGN(B1vp))), P, Real))
         P := RHS(P)
         Gk := 1/B1ik·(B1vp/μt·(LN(P + 1)/LN(P)))^n1·((1 + SIGN(B1vp))/2)
         [n1 := n1, μt := μt, k2 := k2, P := P, Gk := Gk]
```

12AX7 Triode Solutions:

```
#102: TRIODE_Prog(P) := [n1 := 1.352022738, μt := 110.5790947, k2 := 3309.388506, P := 376.6407203, Gk := 686.0117294]
```

The triode and pentode programs run in 15 to 60 seconds on my computer, depending on tube type and data point selection. Some tube types and data point selections may run on and on without giving a result. You may even get complex numbers for results. In these cases, check your data entry numbers first. If the numbers are ok, then try moving data points C1, C2 and D to higher current levels. I can not promise that these programs will work on any tube type, but I think they will work on most.

TRIODE MODEL EQUATIONS:

#103: $$ik = \frac{1}{Gk} \cdot Vt^{n1} \cdot \frac{1 + SIGN(vp)}{2}$$

Where:

#104: $$Vt^{n1} = \sqrt{\left(\left(\left|\frac{Vt}{2}\right| + \frac{Vt}{2}\right) \cdot Vt^{2 \cdot n1 - 1}\right)}$$

Substituting (104) into (103):

#105: $$ik = \frac{1}{Gk} \cdot \sqrt{\left(\left(\left|\frac{Vt}{2}\right| + \frac{Vt}{2}\right) \cdot Vt^{2 \cdot n1 - 1}\right)} \cdot \frac{1 + SIGN(vp)}{2}$$

And:

#106: $$Vt := \frac{vp}{\mu t} \cdot \frac{LN\left(P^{(vg1 + Vct) \cdot \mu t / \sqrt{(k2 + vp^2)} + 1} + 1\right)}{LN(P)}$$

DIODE PARAMETER PROGRAM

For the diode program, data point A1 is placed at the highest current of interest on the diode plate characteristic. Data point A2 is placed at about half the plate voltage of A1.

DERIVE 6 DIODE PARAMETER PROGRAM (5AR4 EXAMPLE)

```
      DIODE_Prog(P) ≔
        Prog
          [Vd ≔]
          [B1ik ≔, B1vp ≔, B2ik ≔, B2vp ≔]
          [n1 ≔]
          B1ik ≔ 0.86
#107:     B1vp ≔ 42
          B2ik ≔ 0.29
          B2vp ≔ 20
          n1 ≔ LN(B2ik·(SIGN(B1vp) + 1)/(B1ik·(SIGN(B2vp) + 1)))/(LN(B2vp) - LN(B1vp))
          Vd ≔ vp
          [n1 ≔ n1, B1ik ≔ B1ik, B1vp ≔ B1vp]
```

5AR4 Diode Solutions:

#108: DIODE_Prog(P) ≔ [n1 ≔ 1.465152649, B1ik ≔ 0.86, B1vp ≔ 42]

DIODE MODEL EQUATIONS:

#109:
$$ik = \frac{B1ik}{B1vp^{n1}} \cdot Vd^{n1} \cdot \frac{1 + SIGN(vp)}{2}$$

Where:

#110:
$$Vd^{n1} = \sqrt{\left(\left(\left|\frac{Vd}{2}\right| + \frac{Vd}{2}\right) \cdot Vd^{2 \cdot n1 - 1}\right)}$$

Substituting (110) into (109):

#111:
$$ik = \frac{B1ik}{B1vp^{n1}} \cdot \sqrt{\left(\left(\left|\frac{Vd}{2}\right| + \frac{Vd}{2}\right) \cdot Vd^{2 \cdot n1 - 1}\right)} \cdot \frac{1 + SIGN(vp)}{2}$$

And:

#112: Vd = vp

DERIVE 6 RESET PROGRAM (This program is for clearing variables so you can make changes to the program. It is not needed to run the program).

#113:
```
RESET(P) ≔
  Prog
    [Vct ≔, Aik ≔, Avp ≔, Avg2 ≔, Avg1 ≔, Vp ≔, Vt ≔]
    [B1ik ≔, B1vp ≔, B1vg1 ≔, B2ik ≔, B2vp ≔, B2vg1 ≔]
    [C1ik ≔, C1vp ≔, C1vg1 ≔, C2ik ≔, C2vp ≔, C2vg1 ≔]
    [Dik ≔, Dvp ≔, Dvg2 ≔, Dvg1 ≔]
    [Evg2 ≔, E1ip ≔, E1vp ≔, E1vg1 ≔, E2ip ≔, E2vp ≔, E2vg1 ≔, E3ip ≔, E3vp ≔, E3vg1 ≔]
    [n1 ≔, μt ≔, μg2 ≔, μp ≔, k2 ≔, P ≔, Gk ≔, a ≔, b ≔, n2 ≔]
```

Ref (1) http://www.normankoren.com/Audio/Tubemodspice_article.html

Ref (2) http://www.knology.net/~billelliott

Bill Elliott email: williamy@knology.net